# Curriculum into the classroom

## Mathematics

## Helpful Information | Number facts: Multiplication and division

### Number facts

Automatic recall of number facts is an essential aspect of efficient mental computation. Other aspects include flexible place value understanding, partitioning and operational sense. Having instant recall of number facts enables students to concentrate on the steps within computation methods.

Using and applying several key thinking strategies flexibly will help students recall number facts efficiently. Many students who are not able to recall key number facts often treat each fact as a new calculation. As a result, students may revert to using inefficient strategies, including extended counting.

Once students have flexible mastery of several key thinking strategies, they will develop secure knowledge of some key number facts and appreciate that from the answer to one problem, other answers can be derived. Strategies for recalling multiplication numbers facts and the related division facts need to be efficient and meaningful.

Students with an understanding of the Commutative principle for multiplication (7 x 3 = 3 x 7) and the Identity principle (multiplying and dividing by one) can greatly reduce the total number of facts that need to be learned.

**Note**: Fluent recall of multiplication and related division facts is anticipated by the end of Year 4.

### Strategies for multiplication facts and related division facts

Curriculum into the Classroom identifies the following thinking strategies for recalling multiplication number facts, and the related division number facts:

- Use doubles
- Use 10
- Use square facts
- Use a rule
- Use a known fact
- Think multiplication for division.

Typically, the thinking strategies for recalling multiplication facts are introduced and rehearsed before an expectation for recalling division facts. For this reason, the following sequence deals with multiplication fact strategies almost exclusively. Related division facts should develop subsequently as students demonstrate fluency with multiplication strategies and an understanding of the connections between multiplication and division.

Students with an understanding of the inverse relationship between multiplication and division will establish an appreciation of related facts. Many students will use their knowledge of related facts to recall division number facts.

The sequence for introducing and teaching thinking strategies for recalling multiplication facts is not pre-determined. It is anticipated that students may be trialling, comparing and practising a range of strategies at any one time. Fluent recall of multiplication and division facts (‘finger click speed’) is anticipated by the end of Year 4.

## Use doubles

- 2s facts – Think ‘addition doubles’, e.g. 6 x 2 as double 6
- 3s facts – Double and add one more factor, e.g. 7 x 3 as ‘double 7, add on 7
- 4s facts – Think ‘double doubles’, e.g. 8 x 4 as ‘double, double 8’ (32)

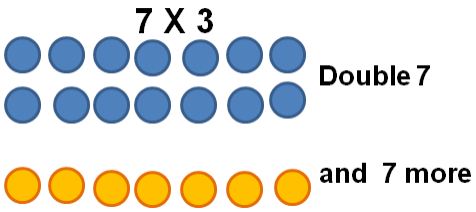
### 2s facts

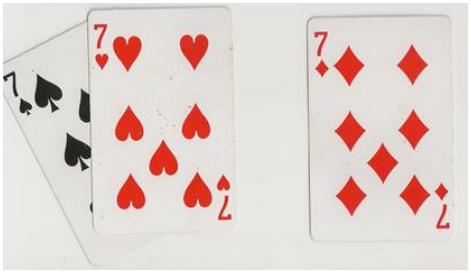
Knowledge of the ‘use doubles’ strategy develops through initial experience with addition doubles. Students will be familiar with common mental images for addition doubles. For example:

The link between addition doubles facts and 2s multiplication facts needs to be made explicitly with students, showing that ‘double 6’ (6 + 6) and ‘double 6’ (2 x 6) are equivalent operations. They can be represented similarly with materials and pictures, but are recorded differently with symbols.

### 3s facts

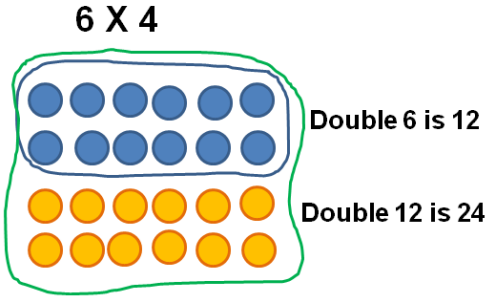
This set of number facts can be represented, visualised and described as ‘double and add one more factor’. For example:

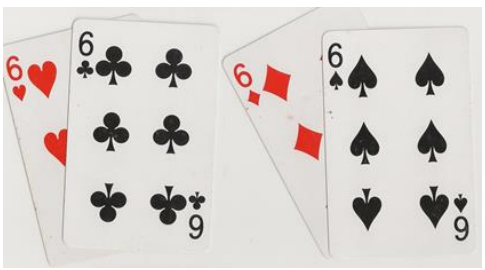


### 4s facts

This set of number facts can be represented, visualised and described as ‘double doubles’. For example:



**Note:** The ‘Use doubles’ strategy can be extended further to include 8s facts (e.g. 6 x 8 as ‘double, double, double 6’).

Some students may find this a reasonable initial strategy. It will generally be replaced by more efficient strategies as students develop fluency (e.g. 6 x 8 as 5 x 8 + 8).

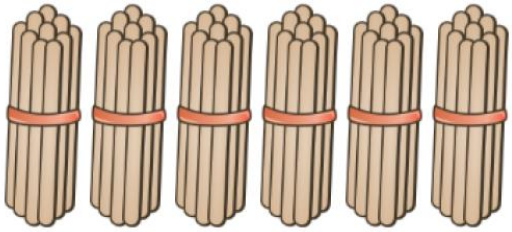
## Use 10

- 10s facts – derived from base ten knowledge (e.g. 6 x 10 = 60)
- 5s facts – Think ‘times 10 and halve’ (e.g. 8 x 5 as 8 x 10 is 80, half of 80 is 40)
- 9s facts – Think ‘times 10 and take away one factor’ (e.g. 8 x 9 as 8 x10 – 8)

## 10s facts

To recall the 10s facts, students use their place value understanding. This set of facts can be represented with structured materials, visualised and described as ‘groups of 10’. For example:

**6 X 10**



**Note**: This representation would be more accurately recorded as 10 x 6 (ten, six times). For the purpose of recalling 10s facts, a flexible application of the commutative principle is helpful.

## 5s facts

In deriving strategies for recalling the 5s facts, students may initially rely on the counting by 5s sequence (e.g. 7 x 5 as ‘5, 10, 15, 20, 25, 30, 35’). Visualising the numerals on an analogue clock face can be a useful cue for recalling the 5s facts for students who are using a counting by 5s strategy.

Students will develop increasingly efficient methods with the 5s facts. These may include:

- multiplying by 10 and then halving (e.g. thinking of 7 x 5 as 7 x 10 (70) and then calculating half (35)). This can prove to be an efficient strategy with larger numbers. (e.g. 68 x 5 as 68 x 10 (680). Half of 680 is 340.)
- halving even factors and multiplying by 10, adjusting for odd factors where necessary, (e.g. 8 x 5 as 4 x 10 and 9 x 5 as 4 x 10 + 5).

Identifying the digit patterns in 5s facts is a significant teaching and learning focus for students using these later strategies. Students may describe the patterns as:

- Even numbers multiplied by 5 will have a product that ends with zero
- Odd numbers multiplied by 5 will have a product that ends with zero.

## 9s facts

One efficient method for recalling the 9s facts is to multiply the factor (multiplicand) by 10 and then subtract the factor from the product (e.g. 7 x 9 as 7 x 10 – 7 (63)).

This can prove to be an efficient strategy for multiplying larger numbers by 9. For example:

**56 X 9** **56 X 10 = 560**

**560 – 56 = 504**

## Use square facts

- Square facts (e.g. 4 x 4 = 16)

Fluency in recalling the square facts can be assisted by visual representations, including arrays made with materials and patterns on a number fact grid.

1 x 1

2 x 2

3 x 3

4 x 4

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 |
| 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| 6 | 0 | 6 | 12 | 18 | 24 | 30 | 36 |

Students will typically use a range of thinking strategies to recall most square facts. Alternative strategies may need to be derived for 6 x 6, 7 x 7 and 8 x 8.

Students may initially work from a known fact to recall these square facts. For example:

| 6 X 6 | 7 X 7 | 8 X 8 |
|---|---|---|
| 6 X 5 = 30 | 5 X 7 = 35 | 4 x 8 is 32 |
| 30 + 6 = 36 | 35 + 14 = 49 | double 32 is 64 |

## Use a rule

- 0s facts – When a number is multiplied by zero, the answer is always zero.
- 1s facts – When a number is multiplied by one, the number is unchanged.

The 0s and 1s facts can be represented meaningfully through grouping and sharing tasks. For example:

2 groups of 4 | 1 group of 4 | 0 groups of 4

**Note**: Students will develop related division facts for 1s facts (e.g. 7 ÷ 1 = 7). However, there are no related division facts for 0s facts.

Simply put, **÷0 facts do not exist**. They cannot be represented meaningfully as there are no real-world contexts for sharing a collection into zero groups. There should be no expectation that students engage with ÷0 facts.

The final two strategies differ from those presented above because they do not relate specifically to any one set of multiplication (or division) number facts. They are valuable strategies that rely on a student's ability to derive personal thinking strategies for solving particular unknown facts.

Typically, these strategies develop after students have developed fluency with using doubles, using 10, using square facts and using a rule.

## Use a known fact

- Derive an answer to an unknown fact from a known fact (e.g. 8 x 6 as 8 x 5 + 8) (48)

Students' use of this strategy needs to be flexible and targeted. Some students will use this strategy frequently, while others who are confident and fluent in a range of more specific strategies may rarely have the need for it.

Using a known fact is helpful in deriving strategies to recall those facts not naturally suited to the earlier strategies. These include the following 'harder facts':

- **6 x 7 (from 6 x 6, e.g. 6 x 6 = 36. Add 6 more... 42)**
- **6 x 8 (from 8 x 5, e.g. 8 x 5 = 40. Add 8 more... 48)**
- **7 x 8 (from 7 x 10, e.g. 7 x 10 = 70. Take away 14... 56)**

Of course, students may choose to work from other known facts, and this should be encouraged. The goal is for students to be flexible and efficient.

## Think multiplication for division

- Think of the related multiplication fact for an unknown division fact (e.g. Solve **24 ÷ 6**, by thinking *'How many 6s in 24?'* 6 x 4 = 24... So, 24 ÷ 6 = 4)

The 'think multiplication' strategy relies on students' understanding of the inverse relationship between multiplication and division and their fluent recall of related facts.

Related facts can be represented with structured materials and models, including basic fact grids, fact family triangles and part-part-whole models.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 |